Enhancing Water Distribution Systems: A Network Interdiction Perspective

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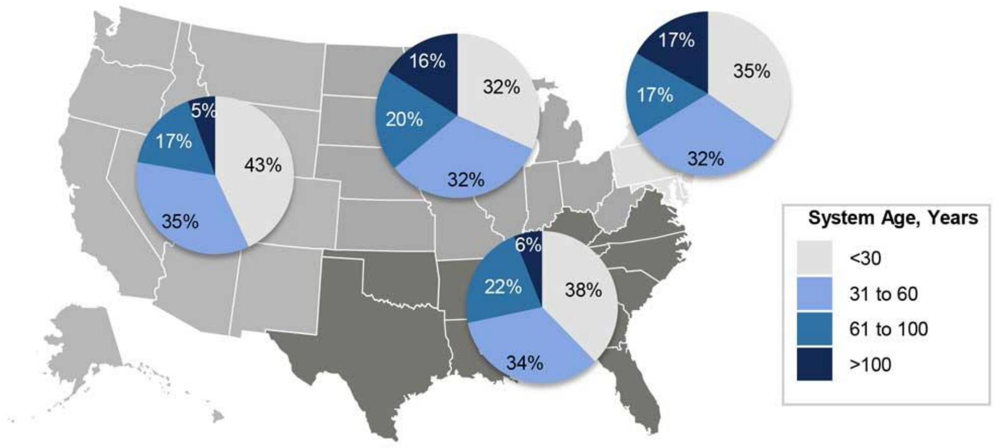
MOTIVATION

- Resilient municipal water infrastructure systems are crucial for distributing potable water throughout the nation everyday
- Unfortunately, America's aging water pipeline networks are highly susceptible to leaks and ruptures



- Disruptions in water distribution systems can have numerous and cascading effects:

- unmet demand
- property damages
- sewage overflow
- contamination of treated water

SIGNIFICANCE OF CONTRIBUTIONS

We aim to

- prescribe optimal sensor placement strategies
- model interdiction (failure of network pipeline components),
- model defense (recovery from the impacts of fortification for a municipal water network)

in order to minimize large-scale disruptions to water distribution networks and prevent the isolation of reservoirs from the rest of a water distribution system.

METHODS

Formulation

MIN — First-level Problem

$$\sum_{i\in[N]} x_i^{(s)} \le b_1, \qquad \forall s\in[S]$$
$$x_i^{(s)} \in \{0,1\}, \qquad \forall i\in[N], \forall s\in[S].$$

- We enforce a maximum on the number of sensors placed in the water pipeline network

MAX — Second-level Problem

$$\sum_{(i,j)\in L} y_{ij} \le k,$$
$$y_{ij} - \tau \le (1-\tau)u_{ij} - \epsilon(1-u_{ij}),$$
$$\sum_{i\in[N]} \ell_i \ge |N| - b_2,$$
$$\sum_{j\in N(i)\cap B_r(i)} x_j(1-u_{ij}) \ge \ell_i, \qquad \forall i\in[N],$$

- We ensure that if an arc's interdiction exceeds as threshold τ, water cannot be conveyed across it
- A minimum number of nodes must be protected via sensors, i.e., a sensor must be placed within a set radius of that node

MIN — Third-level Problem

$$\min_{e,f\ge 0} \frac{1}{S}\sum_{s\in[S]}\left[c_1\sum_{i=1}^{N} e_i^{(s)} + \lambda_2 c_2 \sum_{(i,j)\in L} t^{(s)} + \lambda_3 c_3 \sum_{(i,j)\in L} t^{(s)}\right]$$
$$\text{s.t.} \sum_{i\in[N],(i,j)\in L} (f_{ij}^{(s)} - w_{ij}^{(s)} - p_{ij}^{(s)}) = \sum_{k\in[N],(j,k)\in L} f_{jk}^{(s)} + d_j^{(s)} - e_j^{(s)}, \quad \forall j\in[N], \forall s\in[S],$$
$$-F_{ij}(1-y_{ij}) \le f_{ij}^{(s)}, \quad \forall (i,j)\in L, \forall s\in[S],$$
$$f_{ij}^{(s)} \le F_{ij}(1-y_{ij}), \quad \forall (i,j)\in L, \forall s\in[S],$$
$$t^{(s)} \le f_{ij}^{(s} \quad \forall (i,j)\in L, s\in[S]$$
$$t^{(s)} \ge -f_{ij}^{(s)} \quad \forall (i,j)\in L, s\in[S].$$

- We aim to minimize the total cost of unmet demand, pressure loss, and damage costs
- We enforce flow conservation and capacity constraints

Exact Reformulation

min/max/min → HARD

Dualization →

min/max → BILINEAR

Linearization via McCormick Inequalities →

min/max → LINEAR

Column-Constraint Generation

We iteratively update the upper and lower bounds by determining new interdiction decisions

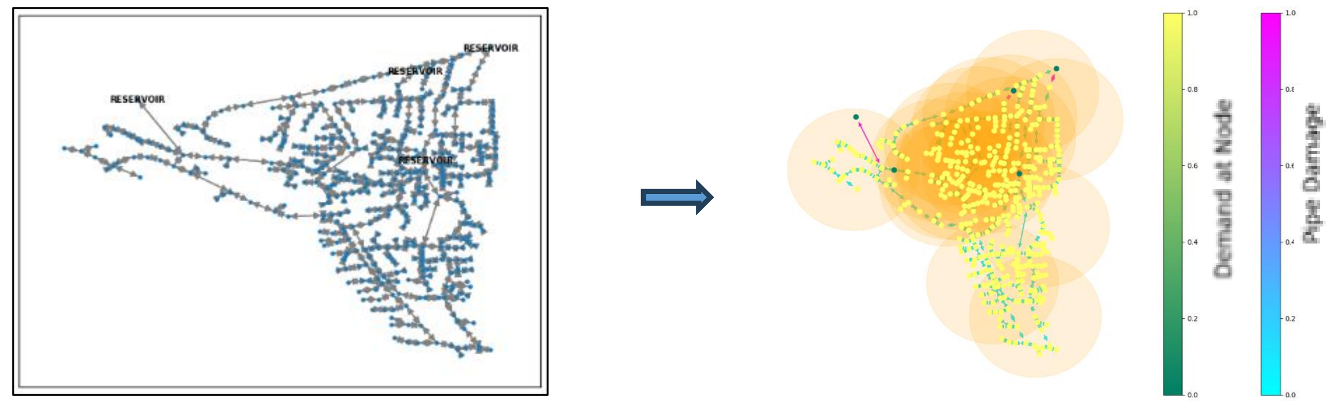
- Let $\hat{Y}_k$ be the collection of interdiction decisions at iteration k and x_k be the fortification decisions at iteration k
- At iteration $k + 1$:
  - Upper Bound: update y_{k+1} with the previous fortification decision x_k and add y_{k+1} to $\hat{Y}_k$ by solving the reformulated max problem
  - Lower Bound: update x_{k+1} with $\{\hat{Y}_k, y_{k+1}\}$

Bender's Decomposition (a subroutine in the column-constraint generation)

- Master Problem: the reformulated inner problem
- Subproblem: the dualized third-level problem
- We solve the primal third-level problem to obtain optimality cuts for the master problem

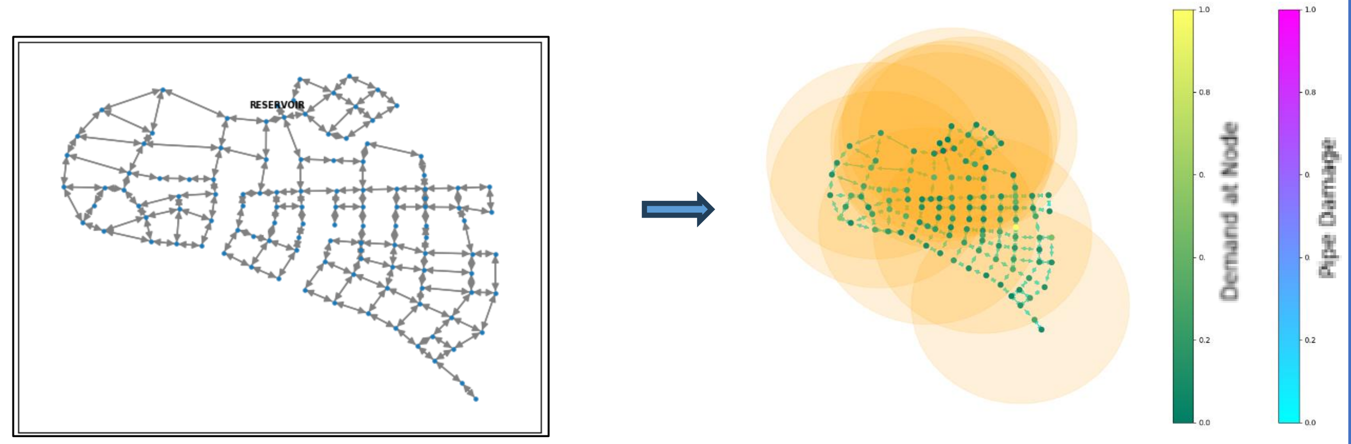
RESULTS

Balerma Network



- Arcs linking 3 out of 4 of the reservoirs to the network have been interdicted
- The 2 reservoirs that are linked to the network via multiple nodes and covered by more than 1 sensors have a lesser chance of being disconnected from the network and causing disruptions throughout the network

ZJ Network



- 2 of the 3 arcs linking the single reservoir to the rest of the nodes in this network are interdicted
- The interdicted pipeline components covered by multiple sensors are well-protected