Enhancing Environmental Monitoring through Optimization: A Faster Branch and Bound Method

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MOTIVATION

To gain a better understanding of a given geographical region's temperature, pollution levels, etc., scientists turn to environmental monitoring using spatially distributed sensors. However, because these sets of sensor data are oftentimes very large, they are proved to be expensive from an efficiency standpoint.

MAXIMUM ENTROPY SAMPLING PROBLEM

To solve this productivity issue, researchers have turned to maximum entropy sampling. Instead of spending time and resources collecting data from every sensor in a set, a smaller subset is selected with the goal of maximizing information, or the entropy, obtained. The differential entropy h(X) of a continuous random variable X with a probability density function f(x) is given by

$$h(X) = -\int_{-\infty}^{\infty} f(x) log f(x) dx$$

In a more mathematical sense, we aim to find the subset s within a covariance matrix C that contains the maximum log determinant.

$$z(C, s) := \max \{\text{ldet} C[S, S] : S \subset N, |S| = s\}$$

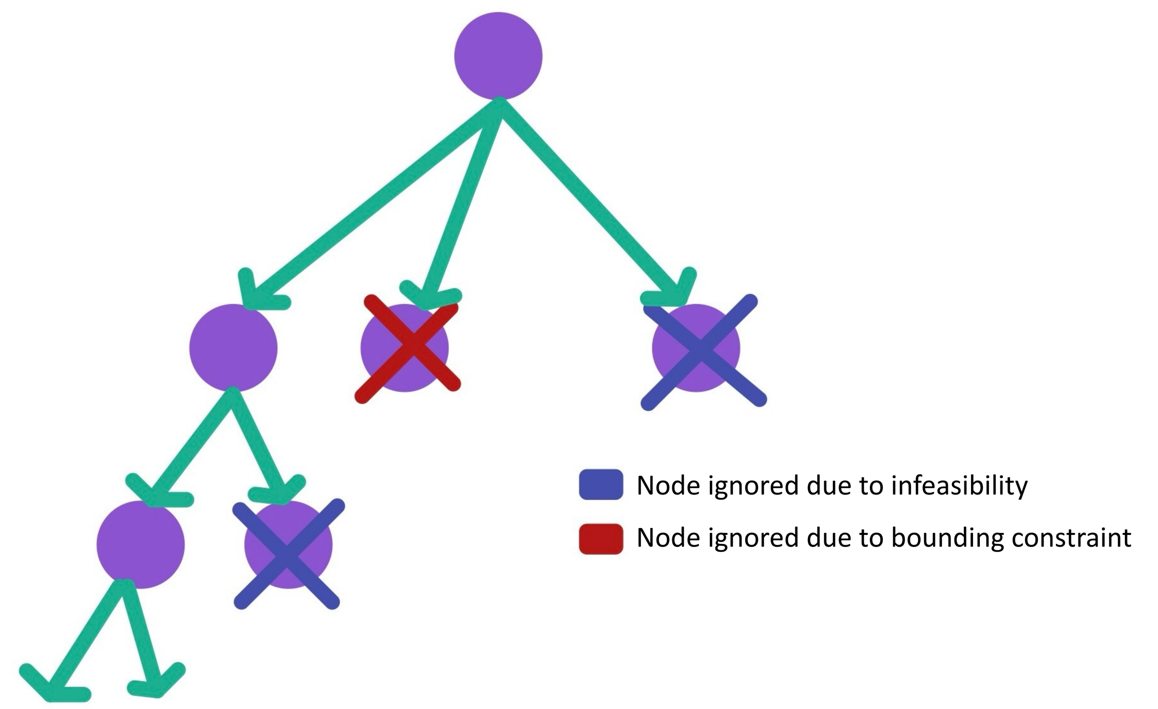
OBJECTIVES

- Improve the MESP solver runtime
  - Tune parameters currently in codebase
  - Implement pruning in branch-and-bound framework
- Reach solutions faster than in previously published work
- Test solver on various subset size instances
- Expand testing to matrices of vastly different sizes

BRANCH AND BOUND FRAMEWORK

- B&B is an important tool that provides an exact solution to a discrete nonconvex problem like MESP
- The general algorithm is as follows:
  - Remove a subproblem within the set of unvisited problems
  - Apply an upper bound (UB) method, in this case Frank Wolfe heuristic algorithm, to obtain UB
  - If current UB ≤ LB (initialized as log determinant of entire covariance matrix), subproblem is discarded
  - If not, a branching index is applied to explore the child nodes
  - Pruning is used to discard subproblems where the solution is greater than the UB
  - Iterations continue until every problem is visited (solved or pruned)

Bounding Diagram



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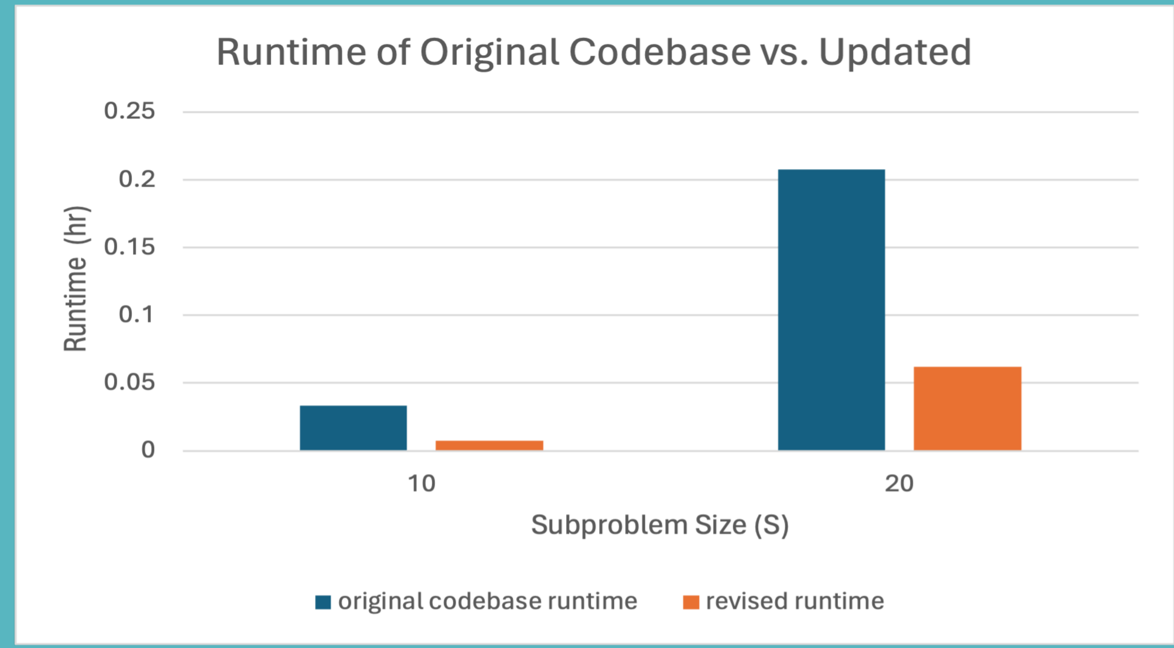
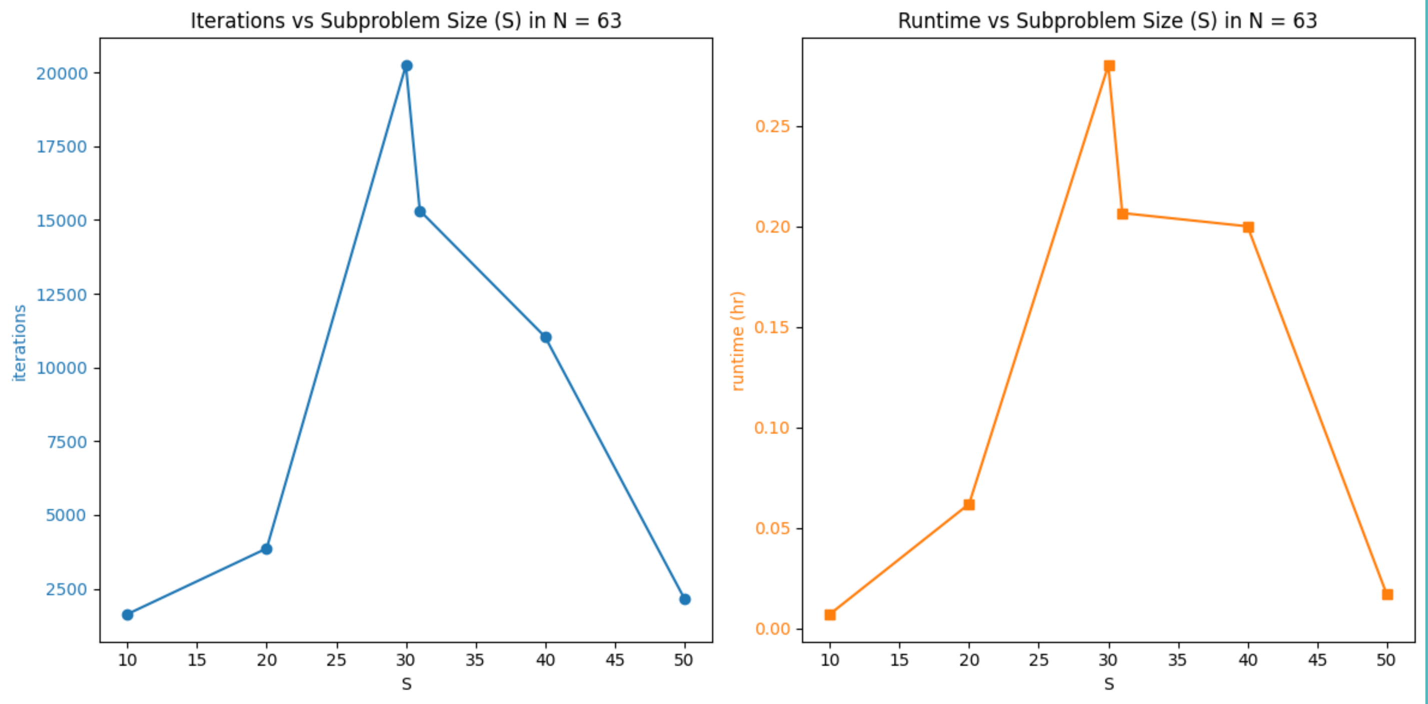
Codebase Repository

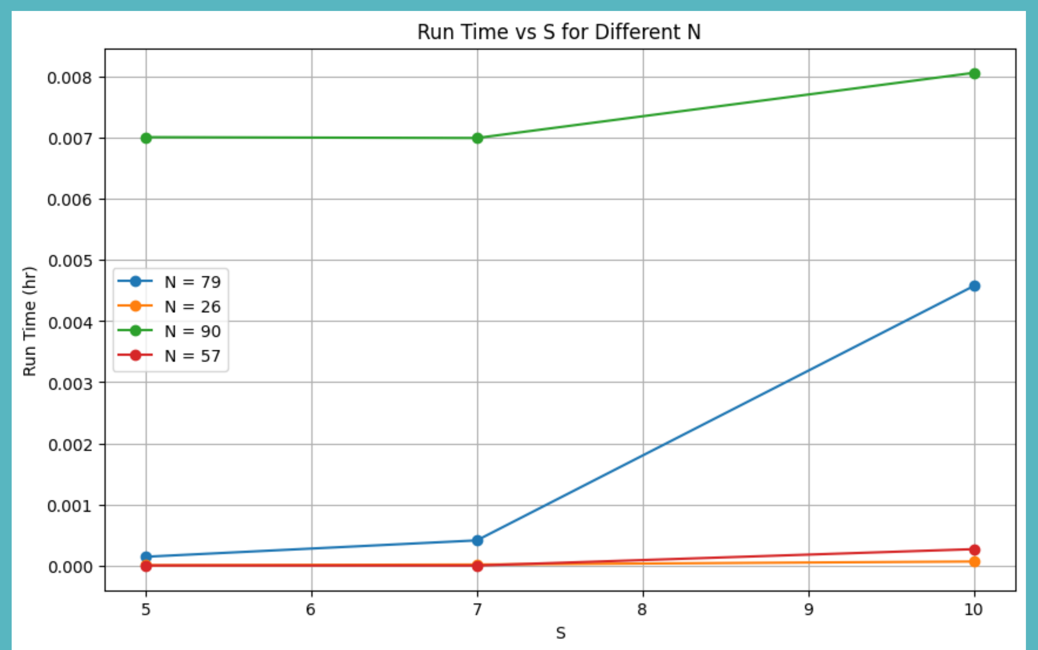
RESULTS



Through the tuning of parameters, such as the upper bound method step size, we see that the runtime of the codebase decreased greatly



Tests on an N=63 dataset showed a proportional relationship between runtime and iterations of the algorithm. It also highlights that the solver's behavior is not linear, as the slowest case tested was s=30.



Expanded testing to several matrices under N =100 show promising results. Even as the subproblem size is increased, we see that the runtime remains to be under a minute.

FUTURE WORK

- Continue to adjust parameters in order to reach faster runtime on all N=63 subproblem instances
- Tune variable-fixing methods in B&B framework to expand the solver's current capabilities to matrices of larger sizes